Ultracehyperbolic Equations: Well-Posedness, Visualization and Applications

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https://www.socr.umich.edu/spacekime/
The default (unrestricted) Cauchy Initial Value problem is ill-posed.

Prescribing values on the characteristic hypersurface $|\kappa| = |x|$ ensures unique solution $|\kappa| \geq |x|$.

\[
\begin{align*}
\Delta_x u(x, \kappa) &= \frac{\Delta_{\kappa} u(x, \kappa)}{\Delta_{d_s} u}, \\
\Delta_x u &= \sum_{i=1} \partial^2_{x_i} u, \\
\Delta_{\kappa} u &= \sum_{i=1} \partial^2_{\kappa_i} u,
\end{align*}
\]

\[
\begin{align*}
u(\kappa_1 = 0) &= u_0(x, \kappa_2), \\
u_{\kappa_1}(\kappa_1 = 0) &= u_1(x, \kappa_2).
\end{align*}
\]
(2D+2D) Visualization of Cauchy Data Solution

Spherical spacekime equation:
A basis Of Separable solution
Visualized at Fixed Spatial Location
Applications

The fMRI data series is intrinsic 3+2 dimensional, which resonates with ultrahyperbolic geometry.

Spatial dimensions: voxels
Temporal dimensions: complex magnitudes

Classification in hyperbolic geometry (SVM)
Whereof one cannot speak, thereof one must be silent.
—Ludwig Wittgenstein