Learning of Network Structure from Neuronal Spike Train Data

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Neural network in brain

Figure 1: *Illustration of neural network in brain*
Neural spike train data

Figure 2: Stem plot of neural spike train data (left) and underlying neural network (right).

- **Goal**: to recover the underlying network structure from the observations of spike train data.
Model-based approach

- Node set (neural ensemble)
  \[ V = \{1, 2, \ldots, V\} \]

- Point process (spike train data)
  \[ T_i = (T_{i,1}, \ldots, T_{i,N_i}), \quad i \in V \]

- Counting process (spike counts)
  \[ N_i(t) = \sum_{\ell=1}^{N_i} I(0 \leq T_{i,\ell} \leq t) \]

- Intensity process (firing rate)
  \[ \lambda_i(t) = \lim_{\Delta_t \downarrow 0} \frac{1}{\Delta_t} P\{N_i(t + \Delta_t) = N_i(t) + 1 \mid \mathcal{F}_t\} \]
Modeling $\lambda_i(t)$

Proposed continuous-time GLM model

$$
\lambda_i(t) = \exp \left\{ \beta_{0,i} + \sum_{j \in \mathcal{V} \setminus i} \beta_{j,i} x_j(t) \right\}, \quad i \in \mathcal{V}, \quad t \in [0, T].
$$

- $\beta_{j,i}$: connection strength parameter.
  - $\beta_{j,i} > 0$: excitatory effect from node $j$ on node $i$;
  - $\beta_{j,i} = 0$: no effect from node $j$ on node $i$;
  - $\beta_{j,i} < 0$: inhibitory effect from node $j$ on node $i$. 

Parameter estimation

\[ \lambda_i(t) = \exp \left\{ \beta_0; i + \sum_{j \in V \setminus i} \beta_{j, i} x_j(t) \right\} := \exp \left\{ \tilde{\beta}_i^T \cdot \tilde{x}_i(t) \right\}. \]

Proposed penalized M-estimator \( \tilde{\beta}_i \)

\[ \tilde{\beta}_i = \arg \min_{\tilde{\beta}_i \in \mathbb{R}^V} \left\{ \mathcal{L}_i(\tilde{\beta}_i) + \mathcal{P}(\tilde{\beta}_i) \right\}. \]

- Loss function: negative log-likelihood function (Ozaki 1979)

\[ \mathcal{L}_i(\tilde{\beta}_i) = -\frac{1}{T} \left[ \int_0^T \log \{ \lambda_i(t) \} dN_i(t) - \lambda_i(t) dt \right]. \]

- Estimated network:

  Excitatory effects \( \hat{E}^+ \) = \( \{ (j, i) : \hat{\beta}_{j, i} > 0 \} \)

  Inhibitory effects \( \hat{E}^- \) = \( \{ (j, i) : \hat{\beta}_{j, i} < 0 \} \)
Real data experiment

- Data: the prefrontal cortex spike train dataset “pfc-6” (of 55 neurons) on CRCNS.

- Estimation result:

Figure 3: Estimated network by our method. Red arrow is excitatory effect, Blue arrow is inhibitory effect.