Ultrahyperbolic Equations: Well-Posedness, Visualization and Applications

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https://www.socr.umich.edu/spacekime/

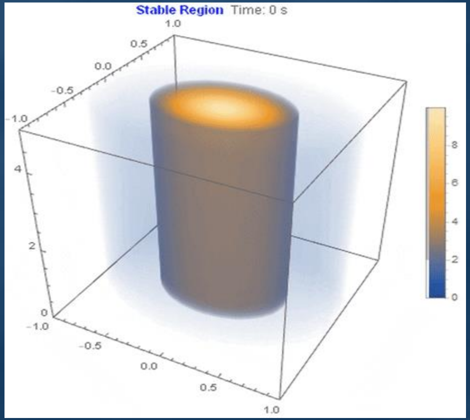
Ultrahyperbolic Wave Equation

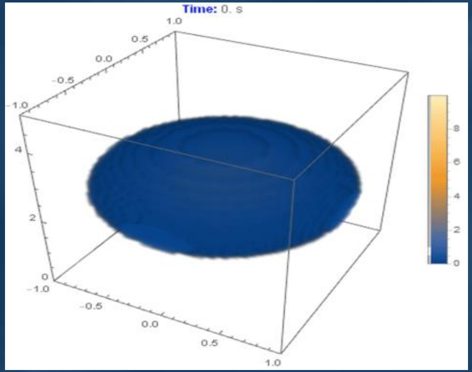
$$\underbrace{\Delta_{\mathbf{x}} u(\mathbf{x}, \boldsymbol{\kappa})}_{\text{spatial Laplacian}} = \underbrace{\Delta_{\boldsymbol{\kappa}} u(\mathbf{x}, \boldsymbol{\kappa})}_{\text{temporal Laplacian}},$$

$$\Delta_{\mathbf{x}} u = \sum_{i=1}^{d_s} \partial_{x_i}^2 u \qquad \Delta_{\boldsymbol{\kappa}} u = \sum_{i=1}^{d_t} \partial_{\kappa_i}^2 u$$

$$\boldsymbol{u}(\kappa_1 = \mathbf{0}) = \boldsymbol{u_0}(\boldsymbol{x}, \kappa_2)$$

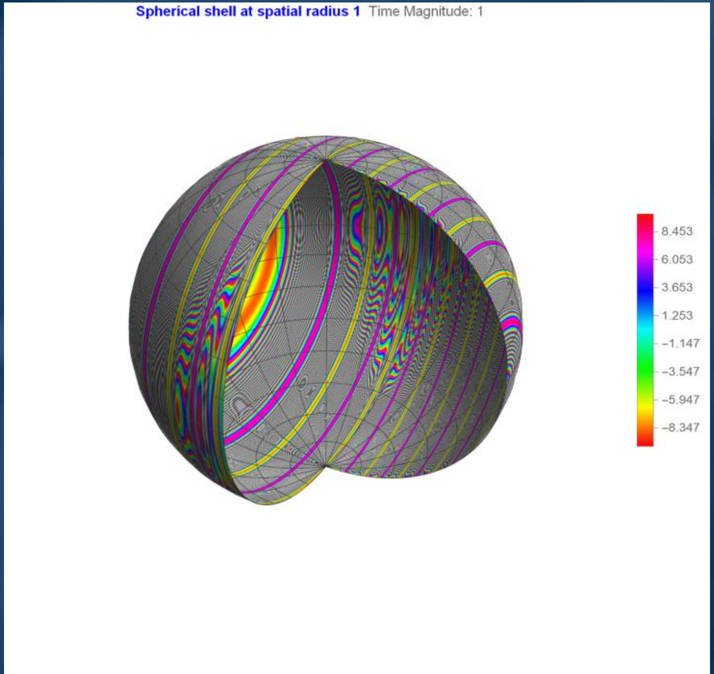
$$\boldsymbol{u}_{\kappa_1}(\kappa_1 = \mathbf{0}) = \boldsymbol{u_1}(\boldsymbol{x}, \kappa_2)$$





- The default (unrestricted) Cauchy Initial Value problem is ill-posed.
- Prescribing values on the characteristic hypersurface|κ|=|x| ensures unique solution |κ|>=|x|

(2D+2D) Visualization of Cauchy Data Solution

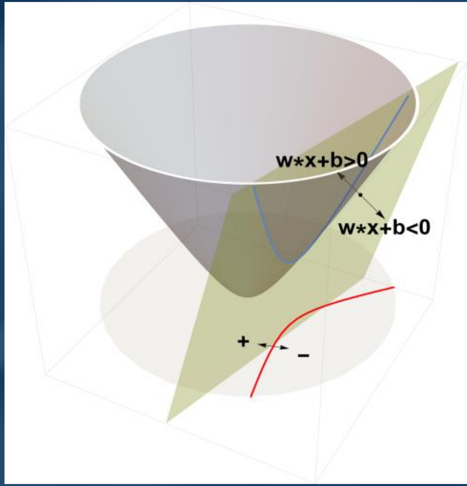


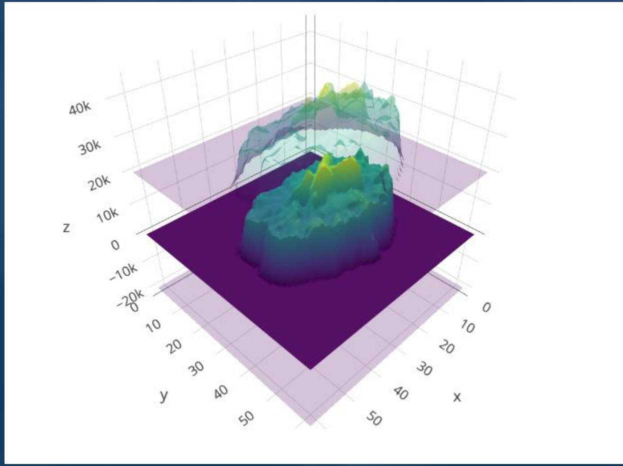
$$\partial_t^2 u + \frac{1}{t}\partial_t u + \frac{1}{t^2}\partial_\phi^2 u = \Delta_x u$$

$$u(1, \phi, x) = f(\phi, x).$$

Spherical spacekime equation:
A basis Of Separable solution
Visualized at Fixed Spatial Location

Applications





Classification in hyperbolic geometry (SVM)

The fmri data series is intrinsic 3+2 dimensional, which resonates with ultrahyperbolic geometry
Spatial dimensions: voxels
Temporal dimensions: complex magnitudes

Whereof one cannot speak, thereof one must be silent.

—Ludwig Wittgenstein